Supplementary Exercises and Exam Questions
Discrete Mathematics with Applications, 3rd Edition
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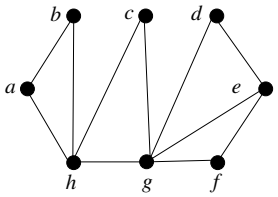
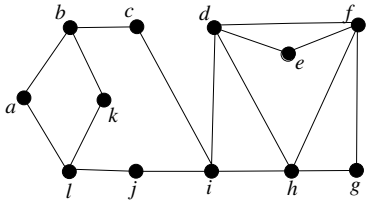
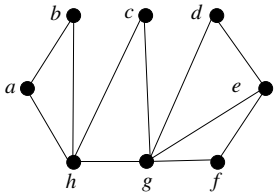
Chapter 11

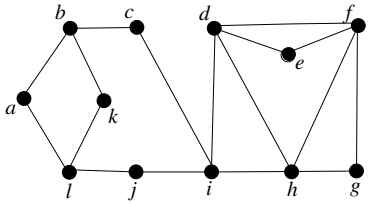
1. If a graph has vertices of degrees 1, 1, 2, 3, and 3, how many edges does it have? Why?
2. For each of (a)–(c) below, either draw a graph with the specified properties or else explain why no such graph exists.
   (a) Graph with six vertices of degrees 1, 1, 2, 2, 2, and 3.
   (b) Graph with four vertices of degrees 1, 2, 2, and 5.
   (c) Simple graph with five vertices of degrees 1, 1, 1, 1, and 5.
3. Determine whether each of the following graphs has an Euler circuit. If it does have an Euler circuit, find such a circuit. If it does not have an Euler circuit, explain why you can be 100% sure that it does not.





4. Determine whether each of the following graphs has a Hamiltonian circuit. If it does have an Hamiltonian circuit, find such a circuit. If it does not have an Hamiltonian circuit, explain why you can be 100% sure that it does not.





5. Draw a directed graph with the following adjacency matrix:

$$\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 2 & 20 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

6. Find the following matrix product:

$$\begin{bmatrix} 2 & 0 \\ 0 & 1 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 1 & 3 & 0 \\ 2 & 4 & 2 \end{bmatrix}$$

7. Consider the adjacency matrix for a graph that is shown below. Answer the following questions by examining the matrix and its powers only, not by drawing the graph. Show your work in a way that makes your reasoning clear.

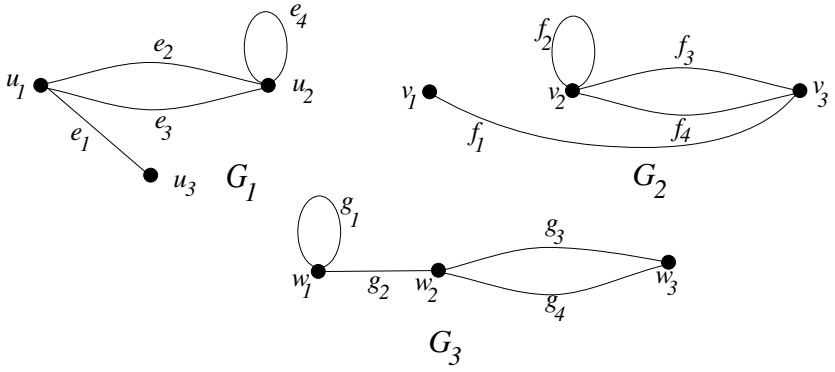
$$\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 2 & 0 \\ 0 & 2 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

(a) How many walks of length 2 are there from v_1 to v_2?

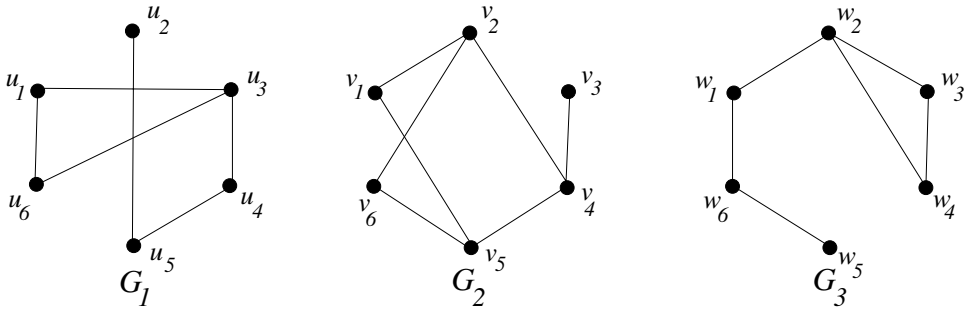
(b) How many walks of length 2 are there from v_1 to v_3?

(c) How many walks of length 2 are there from v_2 to v_2?

8. Determine whether any two of G_1, G_2, and G_3 are isomorphic. If they are, give vertex and edge functions that define the isomorphism. If they are not, give an isomorphic invariant that they do not share.



9. Determine whether any two of the simple graphs G_1, G_2, and G_3 are isomorphic. If they are, give a vertex function that defines the isomorphism. If they are not, give an isomorphic invariant that they do not share.



10. Prove that having a vertex of degree 3 is an invariant for graph isomorphism.

11. A certain graph is 19 vertices, 19 edges, and no nontrivial circuits. Is it connected? Explain.

12. A certain connected graph has 68 vertices and 72 edges. Does it have a nontrivial circuit? Explain.

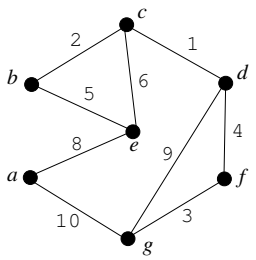
13. Either draw a graph with the given specification or explain why no such graph exists.

(a) full binary tree with 16 vertices of which 6 are internal vertices

(b) binary tree, height 3, 9 vertices

(c) binary tree, height 4, 18 terminal vertices

14. Consider the following weighted graph:



(a) Use Kruskal's algorithm to find a minimum spanning tree for the graph, and indicate the order in which edges are added to form the tree.

(b) Use Prim's algorithm starting with vertex a to find a minimum spanning tree for the graph, and indicate the order in which edges are added to form the tree.