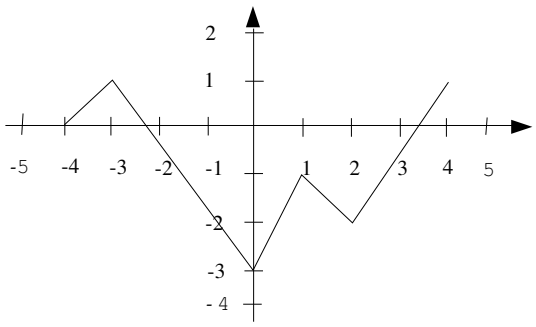
Supplementary Exercises and Exam Questions
Discrete Mathematics with Applications, 3rd Edition
Susanna S. Epp

Chapter 9

1. Draw a careful graph of the function f defined by the formula $f(n) = \left\lfloor \frac{n}{3} \right\rfloor$ for all integers n.

2. Let h be the function whose graph is shown below. Carefully sketch the graph of $2h$.



3. If x is a real number and $x > 1$, is $x^2 > x$? Why? Is $5x^3 > 5$? Why?

4. Consider the statement:

$$3\left|x^2\right| \leq \left|3x^2 + 17x + 5\right| \qquad \text{for all } x > 1.$$

   Express this statement using Ω-notation.

5. Consider the statement:

$$\left|3x^2 + 17x + 5\right| \leq 25\left|x^2\right| \qquad \text{for all } x > 1.$$

   Express this statement using O-notation.

6. Consider the statement:

$$3\left|x^2\right| \leq \left|3x^2 + 17x + 5\right| \leq 25\left|x^2\right| \qquad \text{for all } x > 1.$$

   Express this statement using Θ-notation.

7. Express the following statement using Ω-notation.:

$$\left|x^5\right| \leq \left|\frac{12x^5(3x+4)}{x+2}\right| \qquad \text{for all real numbers } x > 2.$$

8. Express the following statement using O-notation:

$$\left|\frac{12x^5(3x+4)}{x+2}\right| \leq 36\left|x^5\right| \qquad \text{for all real numbers } x > 2.$$

9. Express the following statement using Θ-notation:

$$|x^5| \leq \left|\frac{12x^5(3x+4)}{x+2}\right| \leq 36\,|x^5| \qquad \text{for all real numbers } x > 2.$$

10. Use the definition of O-notation to prove that $2x^2 + 3x + 4$ is $O(x^2)$. (Do not use the theorem on polynomial orders.)

11. Use the definition of O-notation to prove that $15x^3 + 8x + 4$ is $O(x^3)$.(Do not use the theorem on polynomial orders.)

12. Explain why the following statement is true. (You may use the theorem on polynomial orders.)

$$3 + 6 + 9 + \cdots + 3n \text{ is } O(n^2).$$

13. Use the *definition* of O-notation to show that $5x^3 + 3x^2 + 4$ is $O(x^3)$. Be sure to justify each step of your answer.

    (a) Find the total number of additions and multiplications that must be performed when the following algorithm is executed. Show your work carefully.

```
for i := 1 to n
    for j = i to n
        a := 2 · (5 · i + j + 1)
    next j
next i
```

    (b) Find an order for the algorithm segment of part (a) from among the following: $\log_2 n$, n, $n \cdot \log_2 n$, n^2, n^3, and n^4. Give a reason for your answer.

14. (a) Consider the following algorithm segment:

```
for i := 1 to n
    for j := 1 to i
        x := 5 · i + 8 · j
    next j
next i
```

    (b) How many additions and multiplications are performed when the inner loop of this algorithm segment is executed? How many additions and multiplications are performed when the entire algorithm segment is executed?

    (c) Find an order for this algorithm segment from among the following: $\log_2 n$, n, $n \cdot \log_2 n$, n^2, n^3, and n^4. Give a reason for your answer.

15. Describe the operation of the sequential search algorithm.

16. Describe the operation of the insertion sort algorithm.

17. Sketch the graph of $y = \log_3 x$.

18. Define a function F: $\mathbf{R}^+ \longrightarrow \mathbf{R}$ by the formula $F(x) = \log_2(x)$ for all positive real numbers x.

    (a) Graph F, marking units carefully on your axes.

    (b) What is $F(\frac{1}{8})$? Why?

    (c) Write the equation $2^{20} = 1,048,576$ in logarithmic form.

19. If n and k are positive integers and $2^k \leq n < 2^{k+1}$, what is $\lfloor \log_2(n) \rfloor$? Be sure to justify each step of your answer.

20. Use O-notation to express the following statement:

$$| 5x + x \log_2 x | \leq 6 | x \log_2 x | \quad \text{for all } x > 2.$$

21. Describe the operation of the binary search algorithm.

22. Describe the operation of the merge sort algorithm.